inequality induction:

Claim: $\sum_{i=2}^{n} \frac{1}{i^2} \leq \frac{3}{4} - \frac{1}{n}$ for all $n \geq 2$

Base case: $n=2$

$\sum_{i=2}^{2} \frac{1}{i^2} = \frac{1}{4}$ $\qquad \frac{1}{4} \leq \frac{1}{4}$ ✓

$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

IH: Suppose $\sum_{i=2}^{n} \frac{1}{i^2} \leq \frac{3}{4} - \frac{1}{n}$ for $n = 2 \ldots k-1$.

Goal: Show $\sum_{i=2}^{k} \frac{1}{i^2} \leq \frac{3}{4} - \frac{1}{k}$.

$$\sum_{i=2}^{k} \frac{1}{i^2} = \underbrace{\sum_{i=2}^{k-1} \frac{1}{i^2}}_{\text{apply IH}} + \left(\frac{1}{k^2}\right) \leq \frac{3}{4} - \frac{1}{k-1} + \left(\frac{1}{k^2}\right)$$

$$= \frac{3}{4} - \left(\frac{k^2+1-k}{k^3-k^2}\right)$$

I want this $\leq \frac{3}{4} - \frac{1}{k}$

Then I need to show $\frac{k^2+1-k}{k^3-k^2} \geq \frac{1}{k}$

Do not write in solution

$\frac{k^2+1-k}{k^3-k^2} \geq \frac{1}{k}$

$\cancel{k^3} + k - \cancel{k^2} \geq \cancel{k^3} - \cancel{k^2}$

$k \geq 0$ Know this is true.

we know $k \geq 0$

$(+k^3-k^2)$ $(+k^3-k^2)$

$\frac{k+k^3-k^2}{k^3-k^2} \geq \frac{k^3-k^2}{k^3-k^2}$

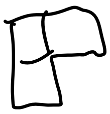
$\frac{k+k^3-k^2}{k^3-k^2} \geq 1$

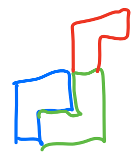
$\frac{1+k^2-k}{k^3-k^2} \geq 1/k$

k and k^3-k^2 are pos $k \geq 2$

so, $\frac{3}{4} - \left(\frac{k^2+1-k}{k^3-k^2}\right) \leq \frac{3}{4} - \frac{1}{k}$, which is what we wanted to show

geometric induction:

 right triomino



Claim: for any pos int n, a $2^n \times 2^n$ sized board/grid with any 1 square missing can be tiled (filled in exactly) using right triominos.

Proof by induction on n.

Base: $n = 1$

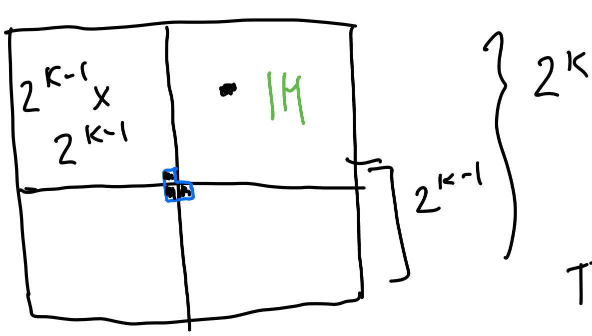
$2^1 \times 2^1$ grid w/ 1 square missing

 true, it's exactly the right triomino

IH: Suppose claim is true for grids $2^n \times 2^n$ w/ 1 square missing for $n = 1 \ldots k-1$

Goal: Show claim is true for $n = k$, or $2^k \times 2^k$ grid

Suppose we have $2^k \times 2^k$ grid with 1 square missing



one $2^{k-1} \times 2^{k-1}$ grid has a missing square, so it can be tiled with right triominos by IH.

Then, we can place a right triomino

in the center of the grid, so the IH can be applied to each quadrant.

Thus, each quadrant can be tiled with right triominos, so the entire grid can too.